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On Soft Compact, Sequentially Compact and Locally Compactness

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Abstract

In this paper, we will defined the concept of soft compact metric space and discuss some results of soft compact metric space. Also we will defined soft locally compact and soft sequentially compact metric space and derived some results on it.

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1. Introduction

Several problems in day today life contains uncertainties and vagueness which can not be solved by classical method. In 1999, Molodstove developed the new concept of soft set to deal with vagueness and uncertainties. AS this area is new and have many applications, many researchers are working in this area. Researchers like Maji, Biswas and Roy [6], M. Shabbir [8], F. Feng [5], Sujoy Das and S.K. Samantha [3,4] have contributed much in the soft set theory.

In this paper we have defined the concept of soft compact metric space and discuss some results of compact metric space. Also we have defined soft locally compact and soft sequentially compact metric space and derived some results on it.

2. Preliminaries

In this section we recall basic definitions and results about soft sets.

Definition 2.1. [7] Let U be the universe and E be the set of parameters. Let $P(U)$ denote the power set of U and A be a non empty subset of E. A pair (F, A) is called a soft set over U where F is given by $F : A \to P(U)$.

In other words, the soft set is a parameterized family of subsets of the set U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the set (F, A), or as the set of ε-approximate elements of the soft set.
i.e. (F, A) is given as consisting of collection of approximations: $(F, A) = \{F(\varepsilon) | \varepsilon \in A\}$.

Definition 2.2. [5] For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is soft subset of (G, B) if

1. $A \subseteq B$ and
2. for all $e \in A$, $\quad F(e) \subseteq G(e)$

and it is denoted by $(F, A)\tilde{\subset}(G, B)$.

Definition 2.3. [5] Two soft sets (F, A) and (G, B) over a common universe U are said to be equal if (F, A) is soft subset of (G, B) and (G, B) is soft subset of (F, A).

Definition 2.4. [5] The complement of a soft set (F, A) over U is denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by $F^c(\lambda) = U - F(\lambda) = F(\lambda)^c$, for all $\lambda \in A$
i.e.$(F, A)^c = \{F(e_i)^c$, for all, $e_i \in A\}$.

Definition 2.5. [6] A soft set (F, E) over U is said to be a absolute soft set denoted by $\tilde{U}$ if $F(e) = U$ for all $e \in E$.

Definition 2.6. [3] Let X be a non empty set and E be a non empty parameter set then the function $\varepsilon : E \to X$ is said to be soft element of X.
A soft element ε is said to belongs to a soft set (F, A) of X if $\varepsilon(e) \in F(e)$, for all $e \in A$ and is denoted by $\varepsilon\tilde{\in}(F, A)$.

Definition 2.7. [3] Let $\mathbb{R}$ be the set of real numbers and $\mathfrak{B}(\mathbb{R})$ be the collection of all non empty bounded subsets of $\mathbb{R}$ and A be a set of parameters. Then the mapping $F : A \to \mathfrak{B}(\mathbb{R})$ is called a soft real set. It is denoted by (F, A). In particular, if (F, A) is singleton soft set then identifying (F, A) with the corresponding soft element, it will be called a soft real number.
We denote soft real numbers by $\tilde{r}, \tilde{s}, \tilde{t}$ and $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in A$ etc.
For example $\bar{0}(\lambda) = 0$ and $\bar{1}(\lambda) = 1$, for all $\lambda \in A$.

Definition 2.8. [3] A soft set (P, A) over X is said to be a soft point if there is exactly one $\lambda \in A$, such that $P(\lambda) = x$, for some $x \in X$ and $P(\mu) = \emptyset$, for all $\mu \in A \backslash \{\lambda\}$. It is denoted by P^x_λ.

Definition 2.9. [3] A soft point P_λ^x is said to belong to a soft set (F, A) if $\lambda \in A$ and $P(\lambda) = \{x\} \subset F(\lambda)$ and we write $P_\lambda^x \tilde{\in} (F, A)$.

Definition 2.10. [3] Two soft points P_λ^x and P_μ^y are said to be equal if $\lambda = \mu$ and $P(\lambda) = P(\mu)$ i.e $x = y$. Thus $P_\lambda^x \neq P_\mu^y$ if and only if $x \neq y$ or $\lambda \neq \mu$.
Let X be an initial universal set and A be a non empty set of parameters. Let $\tilde{X}$ be the absolute soft set. Let $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$. Let $\mathbb{R}(A^*)$ denote the set of all non negative soft real numbers.

Definition 2.11. [4] A mapping $\tilde{d} : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A^*)$ is said to be a soft metric on the soft set $\tilde{X}$ if

1. $\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{\geq} \tilde{0}$ for all $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$,
2. $\tilde{d}(P_\lambda^x, P_\mu^y) = \tilde{0}$ if and only if $P_\lambda^x = P_\mu^y$,
3. $\tilde{d}(P_\lambda^x, P_\mu^y) = \tilde{d}(P_\mu^y, P_\lambda^x)$ for all $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$,
4. $\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{d}(P_\lambda^x, P_\gamma^z) + \tilde{d}(P_\gamma^z, P_\mu^y)$, for all $P_\lambda^x, P_\mu^y, P_\gamma^z \tilde{\in} \tilde{X}$.

The soft set $\tilde{X}$ with the soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$ *or* $(\tilde{X}, \tilde{d})$.

Definition 2.12. [4] Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $\tilde{r}$ be a non negative soft real number. Then the soft set $B(P_\lambda^x, \tilde{r}) = \{P_\mu^y \in SP(\tilde{X}) : \tilde{d}(P_\lambda^x, P_\mu^y) \tilde{<} \tilde{r}\}$ is called soft open ball with center P_λ^x and of radius $\tilde{r}$.

Definition 2.13. [4] Let (Y, A) be a soft subset of metric space $(\tilde{X}, \tilde{d}, E)$. Then the interior of a soft set is denoted as $(Y, A)^\circ$ and is given by

$$(Y, A^\circ) = \{P_\lambda^x \tilde{\in} (Y, A) | P_\lambda^x \tilde{\in} B(P_\lambda^x, \tilde{r}\} \tilde{\subset} (Y, A), \text{ for some } \tilde{r} \tilde{>} \tilde{0}\}$$

Definition 2.14. [4] Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $(Y, A) \tilde{\subset} \tilde{X}$. Then the soft set generated by the collection of all soft points of (Y, A) and soft limit points of (Y, A) in $(\tilde{X}, d)$ is said to be the soft closure of (Y, A) in $(\tilde{X}, \tilde{d}, E)$. It is denoted by $\overline{(Y, A)}$.

Definition 2.15. [9] Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and (Y, A) be a non null soft subset of $\tilde{X}$. Then we say that (Y, A) is soft bounded if there exists $P_\lambda^x \tilde{\in} \tilde{X}$ and a soft real number $\tilde{\epsilon} \tilde{>} \tilde{0}$ such that $(Y, A) \tilde{\subset} B(P_\lambda^x, \tilde{\epsilon})$.

Definition 2.16. [9] Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $(Y, A) \tilde{\subset} (\tilde{X}, E)$. We say that (Y, A) is soft totally bounded if for a given $\tilde{\epsilon} \tilde{>} \tilde{0}$, there exists an $\tilde{\epsilon}$-net for (Y, A) i.e., there exist finitely many soft points $P_{\lambda_i}^{x_i} \tilde{\in} \tilde{X}$ such that $(Y, A) \tilde{\subset} \tilde{\bigcup}_{i=1}^{n} B(P_{\lambda_i}^{x_i}, \tilde{\epsilon})$.

Definition 2.17. [13] Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces. The mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is called a soft mapping, where $f : X \to Y, \phi : E_1 \to E_2$ are two mappings.
The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is one-one soft mapping if $(f, \phi)(P^x_\lambda) = (f, \phi)(P^y_\mu)$ then $P^x_\lambda = P^y_\mu$.
The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is onto soft mapping if $(f, \phi)(X, E_1) = (Y, E_2)$.

Definition 2.18. [13] Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces. The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is said to be soft continuous at the soft point $P^x_\lambda \tilde{\in} SP(\tilde{X})$, if for every $\tilde{\epsilon} \tilde{>} \tilde{0}$, there exists a $\tilde{\delta} \tilde{>} \tilde{0}$ such that for any soft points $P^x_\lambda, P^y_\mu \tilde{\in} \tilde{X}$ with $\tilde{d}_1(P^x_\lambda, P^y_\mu) \tilde{\leq} \tilde{\delta}$,then $\tilde{d}_2\big((f, \phi)(P^x_\lambda), (f, \phi)(P^y_\mu)\big) \tilde{\leq} \tilde{\epsilon}$.

Definition 2.19. [13] Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric spaces. The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is said to be soft uniformly continuous mapping if given any $\tilde{\epsilon} \tilde{>} \tilde{0}$, there exists a $\tilde{\delta} \tilde{>} \tilde{0}$ ($\tilde{\delta}$ depends only on $\tilde{\epsilon}$) for any soft points $P^x_\lambda, P^y_\mu \tilde{\in} \tilde{X}$ with $\tilde{d}_1(P^x_\lambda, P^y_\mu) \tilde{\leq} \tilde{\delta}$,then $\tilde{d}_2\big((f, \phi)(P^x_\lambda), (f, \phi)(P^y_\mu)\big) \tilde{\leq} \tilde{\epsilon}$.

Definition 2.20. [4] A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called soft complete if every soft Cauchy sequence in $\tilde{X}$ converges to some soft point of $\tilde{X}$.

In this chapter, we defined soft compact metric space and discuss some results of soft compact space. Also we defined soft locally compact and derived some results.

3. Soft Compact Metric Space

Definition 3.1. A soft metric space $(\tilde{X}, \tilde{d}, E)$ be a soft metric space. Let $\tilde{\mathbb{C}} = \{(C_i, A_i)\}$ be a family of soft open cover of $\tilde{X}$. Then $\tilde{\mathbb{C}}$ is called a soft open cover of $\tilde{X}$ if each soft point of $\tilde{X}$ is in some (C_i, A_i) in $\tilde{\mathbb{C}}$, that is, $\bigcup_{(C_i.A_i)\tilde{\in}\tilde{\mathbb{C}}} (C_i, A_i) \tilde{=} \tilde{X}$.
A subcollection of $\tilde{\mathbb{C}}'$ of $\tilde{\mathbb{C}}$ whose union is again $\tilde{X}$ then $\tilde{\mathbb{C}}'$ is called soft sub cover of $\tilde{X}$ in $\tilde{\mathbb{C}}$. If $\tilde{\mathbb{C}}'$ is finite, it is called finite soft subcover of $\tilde{X}$ in $\tilde{\mathbb{C}}$.

Definition 3.2. Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric is called soft compact if every soft cover of $\tilde{X}$ has a finite soft subcover f $\tilde{X}$.

Definition 3.3. Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space, and (Y, A) be a non empty soft subset of $\tilde{X}$. Then $\tilde{Y}$ is said to be soft compact in $\tilde{X}$ if $\tilde{Y}$ is soft compact as a subspace of $\tilde{X}$.

Definition 3.4. A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called a soft compact if it is soft complete and soft totally bounded.

Theorem 3.5. Every soft compact metric space is soft soft separable.

Proof. Since every soft compact metric space is soft totally bounded and we know that every soft totally bounded metric space is soft separable. ■

Theorem 3.6. A soft metric space $(\tilde{X}, \tilde{d}, E)$ is soft compact if and only if every soft sequence in $\tilde{X}$ has a soft sequence that converges to a soft point in $\tilde{X}$.

Proof. Let $(\tilde{X}, \tilde{d}, E)$ be a soft compact metric space if and only if it is soft totally bounded and soft complete. Since $(\tilde{x}, \tilde{d})$ is soft totally bounded every soft sequence in $\tilde{X}$ has a cauchy soft sequence and as $(\tilde{X}, d)$ is soft complete. Cauchy soft sequence is convergent. ■

Theorem 3.7. Soft closed subset of a soft compact metric space is soft compact.

Proof. Let $(\tilde{X}, \tilde{d}, E)$ be a compact metric space and that (Y, A) is soft closed in $\tilde{X}$. For any arbitrary soft sequence $(P_{\lambda_n}^{x_n})$ in (Y, A), by above theorem there exists a subsequence of $(P_{\lambda_n}^{x_n})$ that converges to a point $P_\lambda^x \tilde{\in} \tilde{X}$. But as (Y, A) is closed, we should have $P_\lambda^x \tilde{\in} (Y, A)$. Thus, (Y, A) is compact. ■

Theorem 3.8. Soft compact subspace of a soft metric space is closed.

Proof. Let (Y, A) is soft compact subspace of a soft metric space $(\tilde{X}, d, E)$, and let $(P_{\lambda_n}^{x_n})$ be a soft sequence in (Y, A) that converges to a point $P_\lambda^x \tilde{\in} \tilde{X}$. Then, from above theorem, $(P_{\lambda_n}^{x_n})$ has a soft subsequence that converges in (Y, A), and hence we have $P_\lambda^x \tilde{\in} (Y, A)$. Thus, (Y, A) is soft closed. ■

Theorem 3.9. Let $(f, \phi) : (\tilde{X}, d_1, E_1) \to (\tilde{Y}, d_2, E_2)$ be soft continuous. If (H, A) is soft compact in $\tilde{X}$, then $(f, \phi)(H, A)$ is soft compact in $\tilde{Y}$.

Proof. Let $(P_{\mu_n}^{y_n})$ be a soft sequence in $(f, \phi)(H, A)$. Then, $P_{\mu_n}^{y_n} = (f, \phi)(P_{\lambda_n}^{x_n})$ for some soft sequence $(P_{\lambda_n}^{x_n})$ in (H, A). But, since (H, A) is compact, $(P_{\lambda_n}^{x_n})$ has a convergent subsequence, say, $(P_{\lambda_{n_k}}^{x_{n_k}}) \to P_\lambda^x \in \tilde{X}$. Then, since (f, ϕ) is continuous, $(P_{\mu_{n_k}}^{y_n}) = (f, \phi)(P_{\lambda_{n_k}}^{x_{n_k}})$. Thus, $(f, \phi)(H, A)$ is soft compact. ■

Theorem 3.10. Let $(f, \phi) : (\tilde{X}, d_1, E_1) \to (\tilde{Y}, d_2, E_2)$ be soft continuous. If $\tilde{X}$ is soft compact (f, ϕ) is soft closed mapping.

Proof. If (H, A) is a soft closed subset of $\tilde{X}$, then (H, A) is soft compact. Therefore, $(f, \phi)(H, A)$ is soft compact, and thus it is a soft closed set in $\tilde{Y}$. Hence (f, ϕ) is soft closed mapping. ■

Theorem 3.11. Let (f, ϕ) be a one-one, onto and soft continuous mapping from a soft compact metric space $(\tilde{X}, \tilde{d}_1)$ onto a soft metric space $(\tilde{Y}, \tilde{d}_2)$. Then (f, ϕ) is soft homeomorphism.

Proof. To show (f, ϕ) is soft homeomorphism we need to show that $(f, \phi)^{-1}$ is soft continuous. Let (H, A) be a soft closed set in $\tilde{X}$. As $\tilde{X}$ is soft compact, (H, A) is soft compact. Since (f, ϕ) is soft continuous, $(f, \phi)(H, A)$ is soft compact. Therefore, $(f, \phi)(H, A)$ is a soft closed set in $\tilde{Y}$. Since, (f, ϕ) is soft bijective, $\left[(f, \phi)^{-1}\right]^{-1}(A) = (f, \phi)(A)$. ■

4. Soft Locally Compact Metric Space

Definition 4.1. A soft metric space $(\tilde{X}, \tilde{d}, E)$ is said to be soft locally compact if each soft point $P_\lambda^x \tilde{\in} \tilde{X}$ has a soft neighbourhood $B(P_\lambda^x, \tilde{\epsilon})$ such that closure $\overline{B(P_\lambda^x, \tilde{\epsilon})}$ is soft compact.

Example 4.2. Any discrete soft metric space is soft locally compact.

Theorem 4.3. A soft compact metric space is soft locally compact.

Proof. Let $(\tilde{X}, \tilde{d}, E)$ be a soft compact metric space. Then $\tilde{X}$ is soft neighbourhood of any point in P_λ^x in $\tilde{X}$ and the soft closure of this soft neighbourhood $\tilde{X}$ is $\tilde{X}$ itelf which is a compact. ■

Remark 4.4. The converse of this theorem is not true.
An infinite soft discrete space is locally compact but it is not compact.

Proof. Let $B\left(P_\lambda^x, \frac{\tilde{1}}{2}\right)$ be a soft neighbourhood of $P_\lambda^x \tilde{\in} \tilde{X}$, such that its closure $\overline{B\left(P_\lambda^x, \frac{\tilde{1}}{2}\right)} = B\left(P_\lambda^x, \frac{\tilde{1}}{2}\right) = \{P_\lambda^x\}$ is soft compact, being a finite soft subsets of $\tilde{X}$. Hence it is soft locally compact but not soft compact. ■

Theorem 4.5. Let $(\tilde{X}, \tilde{d}_1, E_1)$ be a soft locally compact metric space and $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ be a soft open and soft continuous mapping, then $(f, \phi)(\tilde{X})$ is soft locally compact.

Proof. For any soft point $P_\lambda^x \in \tilde{X}$, there exists a soft set (G, A) containing P_λ^x with compact closure. Then $(f, \phi)(G, A)$ is soft open set in $\tilde{Y}$. Since $(f, \phi)(G, A) \tilde{\cap} (f, \phi)(\tilde{X}) = (f, \phi)(G, A)$ is soft open in $(f, \phi)(\tilde{X})$, and it contains $(f, \phi)(P_\lambda^x)$. Since (f, ϕ) is continuous , $(f, \phi)(\overline{G, A})$ is soft compact. Therefore, $(f, \phi)(P_\lambda^x) \tilde{\in} (f, \phi)(G, A) \tilde{\subseteq} (f, \phi)(\overline{G, A})$. Since $\overline{(f, \phi)(G, A)} \tilde{\subseteq} \overline{(f, \phi)(\overline{G, A})} = (f, \phi)(\overline{G, A})$, it follows that $\overline{(f, \phi)(G, A)}$ is soft compact. Hence $(f, \phi)(\tilde{X})$ is soft locally compact. ■

Theorem 4.6. In a soft locally compact metric space $(\tilde{X}, \tilde{d}, E)$ every non empty soft open set and every non-empty soft closed set is soft locally compact.

Proof. Let (H, A) be a non-empty soft open set in $\tilde{X}$, and P_λ^x be a point in (H, A). There exists a soft open set (G, B) containing P_λ^x such that $\overline{(G, B)}$ is soft compact. Since (G, B) is soft open, there exists a soft real number $\tilde{\epsilon} \tilde{>} \tilde{0}$ such that the soft closed sphere $B_\epsilon[P_\lambda^x] \tilde{\subseteq} (G, B)$. Thus, $B_{\tilde{\epsilon}}[P_\lambda^x] \tilde{\subseteq} \overline{(G, B)}$, and so $B_{\tilde{\epsilon}}[P_\lambda^x]$ is soft compact since it is soft closed subset of a soft compact set. Also, $\exists$ a soft real number $\tilde{\epsilon} \tilde{>} \tilde{\epsilon'} > \tilde{0}$ such that $B_{\tilde{\epsilon'}}[P_\lambda^x]$ is contained in (H, A), and so it is soft compact. Thus (H, A) is soft locally compact.

Let (H, A) be a non-empty soft closed subset of $\tilde{X}$. Let P_λ^x be any soft point in (H, A). Since $\tilde{X}$ is soft locally compact, $\exists$ a soft open set (G, B) containing P_λ^x such that $\overline{(G, B)}$ is soft compact. Since (G, B) is soft open, $(G, B) \tilde{\cap} (H, A)$ is soft open in (H, A). Also $\overline{(G, B)} \cap (H, A)$ is soft closed set in $\tilde{X}$. Since $\overline{(G, B)} \tilde{\cap} (H, A)$ is a soft subset in (H, A). it is a soft closed set in (H, A).

$$\because (G, B) \tilde{\subseteq} \overline{(G, B)}$$
$$cl\big((G, B) \tilde{\cap} (H, A)\big) \tilde{\subseteq} cl\big(\overline{(G, B)} \tilde{\cap} (H, A)\big)$$
$$cl\big((G, B) \tilde{\cap} (H, A)\big) \tilde{\subseteq} \overline{(G, B)} \tilde{\cap} (H, A).$$

Since $\overline{(G, B)}$ is soft compact subset of $\tilde{X}$, $\overline{(G, B)} \tilde{\cap} (H, A)$ is a soft compact subset of (H, A). Thus, $cl\big((G, B) \tilde{\cap} (H, A)\big)$ is soft compact. Hence (H, A) is soft compact. ■

5. Soft Sequentially Compact

Definition 5.1. A soft metric space $(\tilde{X}, \tilde{d}, E)$ is said to be soft sequentially compact if every soft sequence $\{P_{\lambda_n}^{x_n}\}$ in $\tilde{X}$ has a convergent soft sequence $\{P_{x_{n_k}}^{x_{n_k}}\}$.

Theorem 5.2. A soft metric space $(\tilde{X}, \tilde{d}, E)$ is soft sequentially compact if and only if each soft sequence in $\tilde{X}$ has a soft cluster point.

Proof. Let us consider that $\tilde{X}$ s soft sequentially compact. Let $\{P_{\lambda_n}^{x_n}\}$ be a soft sequence in $\tilde{X}$. Let $\tilde{\epsilon} \tilde{>} \tilde{0}$ be a soft real number, $\exists$ a positive integer q such that $\tilde{d}\left(P_{\lambda_{n_k}}^{x_{n_k}}, P_\lambda^x\right) \tilde{<} \tilde{\epsilon} \quad k \geq q$. Let m is any positive integer then choose $r = \max\{n_q, n_m\}$. Then $r \geq q$, $n_r \geq m$ and $\tilde{d}\left(P_{\lambda_{n_r}}^{x_{n_r}}, P_\lambda^x\right) \tilde{<} \tilde{\epsilon}$.

$\Rightarrow$ P_λ^x is a soft cluster point of $\{P_{\lambda_n}^{x_n}\}$.

Conversely, let us suppose that $\{P_{\lambda_n}^{x_n}\}$ be a soft sequence in $\tilde{X}$ having a soft cluster point $P_\lambda^x \tilde{\in} \tilde{X}$. We can choose n_1 such that $\tilde{d}\left(P_{\lambda_n}^{x_n}, P_\lambda^x\right) \tilde{<} \frac{\tilde{1}}{k+}$ continuing this manner there exists a soft sequence $\{P^{n_k}_{x_{n_k}}\}$ of $\{P^{x_n}_{x_n}\}$ that converges to P^x_λ. Therefore this shows that $\tilde{X}$ is soft sequentially compact. ∎

Definition 5.3. A soft metric space $(\tilde{X}, \tilde{d})$ is said to have Bolzano-Weirstrass property if every infinite soft subset of $\tilde{X}$ has a soft limit point in $\tilde{X}$.

Theorem 5.4. A soft metric space is soft sequentially compact if and only if it has the Bolzano-Weirstrass property.

Proof. Let us suppose that $(\tilde{X}, \tilde{d}, E)$ be a soft sequentially compact metric space. Now consider (H, A) be an infinite soft subsets of $\tilde{X}$. We will show that (H, A) has a soft limit point. As (H, A) is infinite soft set, we can form a soft sequence $\{P^{x_n}_{x_n}\}$ from (H, A) of distinct soft points. Since $\tilde{X}$ is soft sequentially compact, $\{P^{x_n}_{\lambda_n}\}$ has a convergent soft sequence $\{P^{x_{n_k}}_{\lambda_{n_k}}\}$ which converges to $P^x_\lambda \tilde{\in} \tilde{X}$. Let $\tilde{\epsilon} \tilde{>} \tilde{0}$ be a soft real number. Then $\exists$ a positive integer m such that $\tilde{d}\left(P^{x_{n_r}}_{\lambda_{n_r}}, P^x_\lambda\right) \tilde{<} \tilde{\epsilon} \quad \forall r \geq m$. This implies $P^{x_{n_r}}_{\lambda_{n_r}} \tilde{\in} B_{\tilde{\epsilon}}\left(P^x_\lambda\right) \quad \forall r \geq n$. Since all the soft points are distinct, the soft open sphere $B_\epsilon(P^x_\lambda)$ contains infinite number of soft points of (H, A).
$\Rightarrow P^x_\lambda$ is a limit point of (H, A).

Conversely, let us suppose that every infinite soft subset of a metric space $(\tilde{X}, \tilde{d})$ has a soft limit point in $\tilde{X}$. Let $\{P^{x_n}_{\lambda_{x_n}}\}$ be a soft sequence in $\tilde{X}$. We show that $\{P^{x_n}_{\lambda_{x_n}}\}$ has a convergent soft subsequence. Consider $(H, A) = \{P^{x_n}_{\lambda_n} | n \in \mathbb{N}\}$ be the soft set of soft points of the soft sequence $\{P^{x_n}_{\lambda_n}\}$. If (H, A) is finite, then some element $\{P^{x_n}_{\lambda_n}\}$ will repeated infinite number of times in the sequence, that is $P^{x_n}_{\lambda_n} = P^{x_m}_{\lambda_m}$ for infinitely many values of n. So, there exists a constant soft subsequence which is convergent. If (H, A) is infinite, then, by our assumption, (H, A) has a soft limit point P^x_λ in $\tilde{X}$. Since P^x_λ is a soft limit point, for each positive integer k, $\exists\, n_k \in \mathbb{N}$ such that $\tilde{d}(P^{x_{n_k}}_{\lambda_{n_k}}) \tilde{<} \frac{1}{2^k}$ then $\{P^{x_{n_k}}_{\lambda_{n_k}}\}$ is a soft subsequence of $\{P^{x_n}_{\lambda_n}\}$ which converges to P^x_λ. Hence $\tilde{X}$ is soft sequentially compact. ∎

Theorem 5.5. Every infinite soft set of soft compact metric space $(\tilde{X}, \tilde{d}, E)$ has a soft limit point in $\tilde{X}$.

Proof. Let $(\tilde{X}, \tilde{d}, E)$ be a soft compact metric space, and (H, A) be an infinite soft subset of $\tilde{X}$. Suppose on contrary (H, A) has no limit point in $\tilde{X}$. Then no soft point of $\tilde{X}$ is a limit point of (H, A). Therefore, for each soft point $P^x_\lambda \tilde{\in} \tilde{X}$, $\exists$ a soft real number $\tilde{\epsilon}_{P^x_\lambda} \tilde{>} \tilde{0}$ such that the soft open sphere $\tilde{B}_{\tilde{\epsilon}_{P^x_\lambda}}(P^x_\lambda)$ contains no point of (H, A) except P^x_λ. Then the collection $\tilde{\mathbb{G}}$, of such soft open spheres is a soft cover of $\tilde{X}$. Since $\tilde{X}$

is soft compact, there is finite subcollection $\{\tilde{B}_{\tilde{\epsilon}^{x_1}_{P_{\lambda_1}}}(P^{x_1}_{\lambda_1}), \tilde{B}_{\tilde{\epsilon}_{P^{x_2}_{\lambda_2}}}(P^{x_1}_{\lambda_2}), \ldots \tilde{B}_{\tilde{\epsilon}^{x_n}_{P_{\lambda_n}}}(P^{x_n}_{\lambda_n})\}$ of $\tilde{\mathbb{G}}$ which covers $\tilde{X}$. Also, we know that each soft open sphere contains at the most one soft point of (H, A), $\bigcup_{i=1}^{n} \tilde{B}_{\tilde{\epsilon}^{x_i}_{P_{\lambda_i}}}(P^{x_i}_{\lambda_i})$ contains only finite number of soft points of (H, A). But $(H, A)\tilde{\subset}\tilde{X}\tilde{=}\bigcup_{i=1}^{n} \tilde{B}_{\tilde{\epsilon}^{x_i}_{P_{\lambda_i}}}(P^{x_i}_{\lambda_i})$ so that (H, A) is finite which contradicts our assumption that (H, A) is infinite soft set. Hence (H, A) contains a soft limit point in $\tilde{X}$. ■

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